# S.5 TERM 1 MATHEMATICS

## SAMPLE ASSESSMENT QUESTIONS

#### ITEM 1

MTN Uganda has been tracking its mobile money subscribers over the past 5 years. The number of active users has been growing exponentially. The company started with 850,000 active users in 2020 and has experienced a constant annual growth rate.

You are a data analyst at MTN Uganda tasked with analyzing this growth to help the company plan for future infrastructure needs. In 2023, the number of active users was recorded as 1,309,500.

#### Tasks:

- a) Utilizing the knowledge of logarithms, determine the annual growth rate of MTN mobile money subscribers. Express your answer as a percentage to 1 decimal place.
- b) Predict the number of active subscribers MTN Uganda will have in 2025.
- c) How many years would it take for the number of subscribers to reach 5 million users?
- d) how can exponential growth models can be used by telecommunications companies for planning resource allocation.

## ITEM 2

Kiira Motors Corporation, Uganda's vehicle manufacturer, is designing components for their new electric vehicle model. The engineers need to calculate exact dimensions for specific components to ensure proper fitment.

You are an engineering intern at Kiira Motors working on the battery bracket design. The chief engineer has given you measurements for a triangular mounting bracket that involves surds. The diagonal support brace of the bracket needs to be exactly  $\sqrt{50}$  cm long. Two sides of the triangular bracket are  $(5 + 3\sqrt{2})$  cm and  $(7 - 2\sqrt{2})$  cm long.

- a) Express the length the bracket diagonal support in the simplified form  $\frac{a}{\sqrt{b}}$ , where a and b are integers and b is not divisible by a perfect square.
- b) Establish the exact length of material needed for both sides combined.
- c) If the perimeter of the entire triangular bracket must be exactly 25 cm, calculate the length of the third side.

A farmer in Mbarara has 800 square meters of land for growing beans and maize. Based on local market prices, she makes a profit of UGX 5,000 per square meter of beans and UGX 3,000 per square meter of maize. Due to crop rotation requirements, she must plant at least 200 square meters of beans and at least 300 square meters of maize. The farmer has approached you he needs help of how he can maximize her profits while meeting all the constraints.

#### Tasks:

- a) Help the farmer to represent all the constraints as inequalities including the total profit
- b) Determine the maximum possible profit and the corresponding areas of land to be allocated for each crop.
- c) If the profit per square meter for maize increases to UGX 4,000, how would this change your recommendation? Show your workings.

#### ITEM 4

The National Water and Sewerage Corporation (NWSC) is designing a water distribution system for three neighboring communities in Kampala. Each community has different water requirements and infrastructure constraints. A consultant engineer working on this project wants to determine the optimal flow rates for each community.

The water distribution system is modeled by the following equations

X + 2Y + Z = 2400 (Total available water supply in liters per minute)

2X + Y + 3Z = 3900 (Pressure balancing equation)

3X + 4Y + 2Z = 5100 (Flow optimization equation)

Where X, Y and Z represent the flow rate in liters per minute in communities A, B and C respectively

The polynomial equation  $P(X) = X^3 - 7X^2 + 14X - 8$  models the operational efficiency of the pumping systems

- a) Help the engineer to determine the optimal flow for each community
- b) If the community A's water requirements increase by 200 liters per minute. What adjustments should be made to other communities' supply to maintain the system balance?
- c) Determine all the possible value X where the efficiency of the pumping system is zero

A small-scale manufacturer in Jinja produces handcrafted furniture. Their workshop makes tables and chairs from locally sourced wood. Each table requires 5 hours of labor and 8 square meters of wood, while each chair requires 3 hours of labor and 2 square meters of wood. The workshop has 60 hours of labor and 72 square meters of wood available per week.

#### **Tasks**

- a) Obtain all the constraints as inequalities that can help the production manager to determine the optimal production strategy.
- b) The profit is UGX 70,000 per table and UGX 25,000 per chair. Model the total weekly profit as a function of the number of table and the number of chairs
- c) Illustrate the feasible region defined by the constraints and use if to find the optimal strategy that maximizes the profit.

#### ITEM 6

The Entebbe Municipal Council is planning to develop a new neighborhood with roads, residential areas, and a community center. The planning department uses a coordinate system where each unit represents 100 meters. An urban planner, needs to design the road network and determine optimal locations for key facilities. Road A is planned to connect points A(2,3) and B(8,15) and another road B will connect points C(4,5) and D(12,9).

- a) Help the urban planner to determine
  - i) The length of the road A in kilometers.
  - ii) The gradient of the road A and the equation that models the path of the road.
- b) Establish whether the two roads A and B intersect, and if so, at what point and the angle between the two roads to the nearest degree.
- c) If a community center needs to be located at a point that is equidistant from points A, B, and C. Find the coordinates of this point

The Uganda National Roads Authority (UNRA) is studying traffic patterns on the Kampala-Jinja highway. They've developed a mathematical model to predict traffic density at different times of the day.

The traffic density function (vehicles per kilometer) at time t hours after 6:00 AM is given by;

$$D(t) = \frac{20t^2 + 30t + 15}{(t^2 + 1)(t + 3)}$$

#### **Tasks**

As a traffic engineer, you need to analyze this model to optimize traffic flow and plan future road expansions.

- a) Representing the function as a sum of partial fractions.
- b) By partial fraction decomposition determine the time of day when traffic density is at its minimum
- c) If the average speed v(t) of vehicles is related to traffic density by the equation  $:v(t) = \frac{80}{D(t)}$ . Determine the time of day when the average speed is maximized\

#### ITEM 8

A renewable energy company is installing solar panels on buildings in Tororo district. To maximize energy collection, they need to determine optimal installation angles based on the building's orientation and the sun's position throughout the year. The company engineer needs to know the ideal mounting angles for a solar project at a school. He finds out that the amount of solar energy collected by the panels is modeled by the function:  $E(\theta) = 800 \sin \theta - 200 \sin 2\theta + 100$  where E is the energy collected in watt-hours per square meter and  $\theta$  is the angle of the panels from the horizontal in radians and the shadow length L of a vertical pole of height h at time t hours after noon is given by:  $L = h \tan \left(\frac{\pi}{12t} + \frac{\pi}{4}\right)$ 

- a) If the school building has a roof with a pitch of 35° facing south. If the optimal angle for solar panels is 15° relative to the roof surface, determine the absolute angle of the panels from the horizontal.
- b) Establish the value of  $\theta$  that maximizes energy collection.
- c) At what time will the shadow be equal to the height of the pole?

The Ministry of Health is analyzing data on malaria cases across different districts in Uganda to allocate resources effectively and plan intervention strategies.

Below is the data on monthly malaria cases reported in 12 Health Centers (HC) in northern Uganda.

Health Centre	Monthly Cases
HC1	145
HC2	187
HC3	203
HC4	168
HC5	227
HC6	192
HC7	254
HC8	176
HC9	219
HC10	238
HC11	182
HC12	209

## **Tasks**

- a) Organize the data into frequency distribution and represent it on a histogram.
- b) Determine;
  - i) The mean number of cases
  - ii) Median number of cases
  - iii) The coefficient of variation
- c) If resources are allocated proportionally to the number of cases, and the total of UGX 60 million is available, how much will be allocated to each category

#### **ITEM 10**

The National Agricultural Research Organization (NARO) is evaluating the performance of a new drought – resistant maize vary across different regions in Uganda. Below is the yield data collected by Agricultural statisticians from 40 farms across the country in tons per hectare.

Yield (tons/ha)	Number of Farms
2.0 - 2.5	3
2.5 - 3.0	7
3.0 - 3.5	12
3.5 - 4.0	10
4.0 - 4.5	5
4.5 - 5.0	3

- a) Illustrate the data on an Ogive and use it to determine the median yield, the interquartile range and the 90<sup>th</sup> percentile.
- b) The standard maize variety has a mean yield of 3.2 tons/ha with a standard deviation of 0.6 tons/ha. Compare the performance of the new variety with the standard variety using appropriate statistical measures.
- c) If farms with yields in the top 20% are selected for seed multiplication, what should be the minimum yield threshold?